

Assessment and Comparison of Two Source Capture Recapture Methods Performance

İki Kaynak Yakalama Tekrar Yakalama Yöntemlerinin Performansının Değerlendirilmesi ve Karşılaştırılması

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ABSTRACT Objective: The aim of this study is to assess the performance of Lincoln – Petersen, Chapman and Bailey two source Capture Recapture methods. **Material and Methods:** The Capture Recapture methods were applied with simulation studies on data obtained from survey to estimate the population size of families with at least one disabled individual in Turkey with small, medium and large samples, and to compare the performance of these estimators with different sample sizes with Root Mean Squared Error (RMSE) measure. **Results:** From the comparison part of the mentioned Capture Recapture methods, the Lincoln - Petersen method provides more reliable results than Chapman and Bailey methods in small sample size one and two, but undefined when there were no element recaptured in sample two. Chapman and Bailey methods provides better estimates than Lincoln – Petersen method when dealing with big sample one and small sample two. When working with big samples are all the three estimators mentioned in this study provides better estimates with least RMSE. **Conclusion:** At the conclusion part of this study we remark that, when dealing with small samples I and II Lincoln – Petersen should be considered but when no element recaptured in the second sample researchers should consider Chapman and Bailey methods as alternative. When dealing with big size sample I and small size sample II Chapman and Bailey methods should be considered and when both samples are large all the methods discuss in this study can be considered.

Keywords: Capture recapture; assessment and comparison; lincoln-petersen method; chapman method; bailey method

ÖZET Amaç: Bu çalışmanın amacı, Lincoln – Petersen, Chapman ve Bailey Capture Recapture yöntemlerinin performanslarını simülasyon çalışmaları ile değerlendirmektir. **Gereç ve Yöntemler:** Bu çalışmada, Türkiye’de küçük, orta ve büyük örneklemlemler ile en az bir engelli birey içeren ailelerin popülasyon büyüklüğünü belirlemek için yapılan anket verileri kullanılmıştır. Tahmin edicilerin performanslarını Hata Kareler Ortalamasının Kökü (RMSE) ölçütü kullanılarak karşılaştırılmıştır. **Bulgular:** Yakalama Tekrar Yakalama yöntemlerinden olan Lincoln - Petersen yöntemi, Küçük örneklemlemler büyüklüğü Chapman ve Bailey yöntemlerine göre daha güvenilir sonuçlar vermektedir, ancak geri yakalama (Recapture) değeri sıfır olduğunda tanımsızdır. Büyük örneklem bir ve küçük örneklem iki olduğunda Chapman ve Bailey yöntemleri Lincoln-Petersen yönteminden daha iyi tahmin sonuçları vermiştir. Her iki örneklem büyüklüğü büyük olduğunda ise, bu çalışmada bahsedilen üç tahmin yöntemlerinin hepsini düşük RMSE ile iyi tahmin sonuçları vermiştir. **Sonuç:** Bu çalışmanın sonucunda, küçük örneklem bir ve iki ile ilgilenildiğinde Lincoln-Petersen yöntemi, fakat ikinci örneklemde hiçbir birim tekrar yakalanmadığında Chapman ve Bailey yöntemlerini alternatif olarak düşünmemelidir. Büyük boyutlu örneklem bir ve küçük boyutlu örneklem iki ile ilgilenilirken Chapman ve Bailey yöntemleri dikkate alınmalı ve her iki örnek büyük olduğunda bu çalışmada tartışılan yöntemlerin tümü düşünülebilir.

Anahtar Kelimeler: Yakalama tekrar yakalama; değerlendirme ve karşılaştırma; lincoln- petersen yöntemi; chapman yöntemi; bailey yöntemi

Census is the process of reaching all the elements in a population and obtaining the desired information.¹ As census is considered as means of assessing population sampling survey methods are also important in assessing population size. One of the statistical survey methods used to estimate a population size is Capture recapture method. Capture recapture methods have been applied to estimate population size when the number of elements in a population is unknown.²

Despite Capture recapture methods were been widely used in estimating the perimeters of fish and wild animal abundance, in recent years the methods are among the common statistical methods applied in health sciences, epidemiology and social fields in estimating population size.^{3,4} Besides estimating population size, Capture recapture methods are been use to assess population parameters such as varieties of wild animals, immigration, emigration and survival rate.⁵

In Capture Recapture method, samples are taken in two or more occasion at random from the study population, the first sample caught are release to the population after marking or tagging. Capture Recapture methods are applied by counting the number of tagged or marked elements in each sampling occasion after the first sample.⁶

The Capture Recapture methods are categorized in to open and closed depending on the structure of the population. In some cases the population will not be closed for increase or decrease in terms of birth, death, migration and other factors such data source is classified as open source. While an open population is one that changes during the study period, because of any combination of births, deaths, immigration, or emigration.⁷ Capture Recapture methods for closed populations are mainly grouped in two categories: those dealing with only two samples and those dealing with more than two samples.^{3,8,9} This study is designed to focus on assessing the performance of Lincoln – Petersen, Chapman and Bailey two source Capture Recapture methods on different sample sizes.

CAPTURE RECAPTURE METHODS FOR CLOSE DATA SOURCE

Close population methods assume that there are no gains (births or immigration) or losses (deaths or emigration) of element during the course of the study.¹⁰

The basic assumptions of two source Capture Recapture methods are:

- (I) The closure of list or study field to increased or decreased: in such case the population is assumed closed to increase or decreased throughout the study.
- (II) All elements have equal chance of appearing in each sample: in such case the tag or mark should not affect the inclusion of the element after the first sample and all elements will have equal chance of appearing in the samples.
- (III) Marks or tagging given to the elements will remain: the marks or tags on the elements are assumed to be on the elements for identification throughout the study period.
- (IV) The independency of the data source: all the samples assumed to be independently taken from the study population.^{1,2}

CAPTURE RECAPTURE METHODS FOR TWO SOURCES CLOSED POPULATION

The simplest version of this methodology, also known as dual systems estimation which involves only one marking and two sampling occasions is the two source Capture Recapture and it is only applicable for close population. Some of the points to be considered for these methods are: when elements lost in the population, estimate will be for the beginning of the study, when elements gained during the study period, the estimated population size will be for after the first sample and when there are gains and losses of elements the estimates will be biased.¹⁰

APPLICATIONS OF TWO SOURCE CAPTURE RECAPTURE METHODS IN FIELD STUDY

Capture Recapture methods are applied in field survey to estimate the population size of specific animals, homeless people etc. The Capture Recapture methods for two sampling occasion is applied following the procedures given bellow.

- i. Randomly n_1 elements are captured as first sample from the study population, after the first sample n_1 marked and released in to the population, the proportion of marked elements in the population will be n_1/N , where N is the population size.⁴
- ii. After given time for the marked to mix with the unmarked elements in the population, second sample n_2 is captured and in sample n_2 the mark elements are recorded as m_2 , contingency table for two source Capture Recapture methods is given in Table1.

APPLICATIONS OF TWO SOURCE CAPTURE RECAPTURE METHODS IN STUDIES WITH TWO REGISTRY LISTS

Capture Recapture methods are applied in registry list to estimate the population size of interest following the steps given bellow. This list might be patients in a hospital, employees in a firm etc.

- i. Two lists are collected as list n_1 , list n_2 and matched to identify the elements appeared in both lists.
- ii. For easy identification of the elements, demographic or other identities like name, surname, sex, age, house number etc. are used to identify the matching elements. Then m_2 will donate the matching elements identified from the lists.

Two by two table is formed as given in Table 1 with n_1 number of elements in first sample, n_2 number of elements in second sample, m_2 number of matching element captured in both samples, (n_1-m_2) number of elements in first sample not in the second sample, (n_2-m_2) number of elements found in second sample not in first sample, N total number of elements in the population.

The number of elements that did not appear in either of the samples (m_1) is unknown, therefore will be estimated by m_1 using independency in equation-1

$$\hat{m}_1 = \frac{(n_2-m_2)(n_1-m_2)}{m_2} \tag{1}$$

The independency assumption in Capture Recapture studies is tested with the formula given in equation-2

$$\frac{m_1 m_2}{(n_2-m_2)(n_1-m_2)} \approx 1 \tag{2}$$

If the two sources were independent the odds close to one.

LINCOLN-PETERSEN METHOD

Lincoln – Petersen method is applied to estimate the population size of unknown closed population by sampling in two different occasions.⁷

Lincoln – Petersen method is applied when a population is closed to increase or decrease, all elements have equal chance of capturing in each sampling occasion and when the two samples are independent.¹¹

TABLE 1: Contingency table for two source Capture Recapture methods.

Samples		Sample 2		
		Present	Absent	Total
Sample 1	Present	m_2	n_1-m_2	n_1
	Absent	n_2-m_2	m_1	
	Total	n_2		N

In Lincoln – Petersen experiments, after the first sample n_1 marked and released in to the population, the proportion of marked elements in the population will be n_1/N and when the second sample n_2 elements are captured the proportion of the marked elements in the sample n_2 will be m_2/n_2 . If the two samples are independent, then number of the elements recaptured should be approximately equal to the marked rate in the population.³ Therefore we have,

$$\frac{n_1}{N} = \frac{m_2}{n_2} \tag{3}$$

From equation-3 the Lincoln-Petersen estimator of total population size (\hat{N}_{LP}) is given in equation-4;

$$\hat{N}_{LP} = \frac{n_1 n_2}{m_2} \tag{4}$$

Variance and standard deviation of the total population is given by Lincoln – Petersen are given equation-5 and equation-6 respectively;

$$\text{Var } \hat{N}_{LP} = \frac{n_1 n_2 (n_1 - m_2)(n_2 - m_2)}{m_2^3} \tag{5}$$

$$\text{sd } \hat{N}_{LP} = \sqrt{\text{Var}(\hat{N}_{LP})} = \sqrt{\frac{n_1 n_2 (n_1 - m_2)(n_2 - m_2)}{m_2^3}} \tag{6}$$

Confidence interval of Lincoln – Petersen is given in equatoin-7.

$$\text{CI}_{\hat{N}_{LP}} = \hat{N}_{LP} \pm Z_{\alpha/2} \times \text{sd } \hat{N}_{LP} = \hat{N}_{LP} \pm Z_{\alpha/2} \sqrt{\frac{n_1 n_2 (n_1 - m_2)(n_2 - m_2)}{m_2^3}} \tag{7}$$

CHAPMAN METHOD

Lincoln–Petersen method is undefined when the number marked elements captured in the second sample is zero ($m_2=0$), i.e., when there is no element appeared in both sample one and sample two. This problem was addressed by Chapman by finding a modification of the Lincoln– Petersen Index able to estimate the population size also when $m_2=0$.¹²

Chapman developed a modification of Lincoln – Petersen method to overcome the problems when the number of elements recaptured in second sample is zero $m_2=0$. The Chapman method depends on Hypergeometric model which assumed n_1 and n_2 to be constant.

The population size is estimated by Chapman method as given in equation-8

$$\hat{N}_{CP} = \frac{(n_1+1)(n_2+1)}{(m_2+1)} - 1 \tag{8}$$

Variance and standard deviation of the population estimated by Chapman is given in equation-9 and equation-10 respectively

$$\text{Var}(\hat{N}_{CP}) = \frac{(n_1+1)(n_2+1)(n_1-m_2)(n_2-m_2)}{[(m_2+1)^2(m_2+2)]} \tag{9}$$

$$\text{Sd}(\hat{N}_{CP}) = \sqrt{\frac{(n_1+1)(n_2+1)(n_1-m_2)(n_2-m_2)}{[(m_2+1)^2(m_2+2)]}} \tag{10}$$

Confidence interval of Chapman Method is given in equation-11.

$$CI_{\hat{N}_{CP}} = \hat{N}_{CP} \pm Z_{\alpha/2} \times sd_{(\hat{N}_{CP})} = \hat{N}_{CP} \pm Z_{\alpha/2} \sqrt{\frac{(n_1+1)(n_2+1)(n_1-m_2)(n_2-m_2)}{[(m_2+1)^2(m_2+2)]}} \quad (11)$$

BAILEY METHOD

Lincoln – Petersen method of population estimation is unbiased when the number of elements recaptured in the second sample (m_2) is big enough (greater than 20), but when m_2 is small the method is biased and undefined when $m_2=0$.¹³ Bailey developed a modification of Lincoln – Petersen method by 1 to second sample n_2 and marked elements recaptured in the second sample m_2 which is said to be less bias.¹⁴

The population size is estimated by Bailey method as given in equation-12

$$\hat{N}_{BL} = \frac{n_1(n_2+1)}{(m_2+1)} \quad (12)$$

Variance and standard deviation of the population estimated by Bailey is given in equation-13 and equation-14 respectively

$$\text{Var}_{\hat{N}_{BL}} = \frac{n_1^2(n_2+1)(n_2-m_2)}{(m_2+1)^2(m_2+2)} \quad (13)$$

$$sd_{\hat{N}_{BL}} = \sqrt{\frac{n_1^2(n_2+1)(n_2-m_2)}{(m_2+1)^2(m_2+2)}} \quad (14)$$

Confidence interval of Bailey Method is given in equation-15

$$CI_{\hat{N}_{BL}} = \hat{N}_{BL} \pm Z_{\alpha/2} sd_{\hat{N}_{BL}} = \hat{N}_{BL} \pm Z_{\alpha/2} \sqrt{\frac{n_1^2(n_2+1)(n_2-m_2)}{(m_2+1)^2(m_2+2)}} \quad (15)$$

MATERIAL AND METHODS

The aim of this study is to assess the performance of two source Capture Recapture methods namely Lincoln – Petersen, Chapman and Bailey methods on different sample sizes. Data for this study was obtained from survey conducted on Uludag University students using Capture Recapture techniques aimed to estimating the population size of families with at least one disabled individual. The survey questionnaire for this study is approved by Uludag University ethics Board (Grant no: 2015-13/5).

QUESTIONNAIRE FOR APPLICATION OF CAPTURE RECAPTURE STUDY

Sample Size

The population size of the student studying at Uludag University Gorukle campus at the time of the study was $N = 38258$ and disability rate in Turkey by other study is 0.026.¹⁵ The margin of error $d=0.01$ with significance level $\alpha=0.05$. This information was considered as reference for the calculation of sample size needed in this study, which gave us the sample size needed in each sampling occasion I and II ($n=1000$).

The Procedures Followed in Capture Recapture Application;

The basic assumptions for applying two sources Capture Recapture methods are independency of samples, each element has equal chance of being captured or appeared in each sample and the list or study field must be closed to increase or decrease during the study period. Uludag University Gorukle campus, 2015-2016 academic session, stream semester was considered suitable to conduct the study.

The samples were taken from 5 different locations in the campus (Library bus top, Hostel bus stop, auditorium bus stop, cafeteria bus stop and university metro station) by same pair of survey experts in each

sampling occasion and location. Each of the two samples was taken in ten working days and three weeks interval was given between the samples.

For easy identification of the recaptured individuals, some socio demographic information were asked and in the second sample survey questionnaire “have you respond to this questionnaire before (in the past three weeks)” question was added.

ASSESSMENT AND COMPARISON OF LINCOLN – PETERSEN, CHAPMAN AND BAILEY METHODS ON DIFFERENT SAMPLE SIZE

Number of number of extended families with disabled individual found in the study survey is 528 out of 1953 responders was considered as hypothetic population of this study. Second samples of the study were drawn equal or less than the first sample size at random from the hypothetic population considered for this study in 25, 50, 100, 250, 500, 750 and 1000. For each of the Capture Recapture combination, 200 replicate were performed. The sample was repeated when the result is undefined to apply Lincoln-Petersen methods. Root Mean Square Error (RMSE) measure was used to compute the performance of the Lincoln - Petersen, Chapman and Bailey methods. (Equation-16)

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (\hat{N}_i - N)^2}{n}} \quad (16)$$

\hat{N}_i : Estimated population i

N: True population of the study

n: Number of replicates

FINDINGS

COMPARISON OF LINCOLN – PETERSEN, CHAPMAN AND BAILEY METHODS BY SAMPLE SIZE

Different of sample size I and II were used to evaluate and compare the performance of Lincoln – Petersen, Chapman and Bailey Methods with RMSE given in Table 2.

RESULTS AND DISCUSSIONS

Population census becomes difficult due to time consuming, large amount of budget and other relevant factors. Sampling survey methods are applied when census becomes impossible. Population size is important either in getting information of the study population sampling or interpreting the results and findings of study on population. In some cases we may interested in knowing the population of specific elements or elements with specific features and we cannot take a complete census to get the information. In such cases Capture Recapture techniques are designed to formulate estimates of population size.¹⁶ In this study we focused on assessing the performance and comparison of the performance of three Capture Recapture methods namely Lincoln - Petersen, Chapman and Bailey Methods used in estimating population size of closed population.

In estimation of population of this study, Lincoln - Petersen gave better estimate than Chapman and Bailey methods when the first sample size is small. However, when the number of recaptured elements is not big enough (greater than 20), the estimate of the population could be biased.^{13,18} When the first sample size is small and second sample size is increasing, Lincoln – Petersen showed improvement. When comparing Chapman and Bailey in small size sample one, Chapman method showed better estimate even though there were no big differences (Figure 1).

TABLE 2: Root Mean Square Errors of the estimators.

Sample I	Sample II	Lincoln - Petersen	Chapman	n1 Bailey
C50	R25	431.82	469.06	473.64
	R50	358.62	426.60	432.42
C100	R25	354.60	424.13	428.54
	R50	257.29	354.51	359.46
	R100	228.54	240.63	248.71
C250	R25	284.33	323.00	324.66
	R50	267.37	204.98	207.58
	R100	243.60	159.58	161.53
	R250	192.87	140.70	139.05
C500	R25	301.16	189.92	190.54
	R50	274.34	164.22	164.42
	R100	136.81	117.93	117.43
	R250	73.56	70.16	69.88
	R500	53.15	52.22	52.00
C750	R25	368.99	186.00	185.56
	R50	283.12	180.12	179.51
	R100	148.00	124.62	124.30
	R250	94.11	88.41	88.12
	R500	59.89	57.84	57.44
	R750	46.69	45.25	44.69
C1000	R25	314.86	172.96	172.51
	R50	170.54	137.80	137.30
	R100	123.44	104.97	104.62
	R250	70.18	66.73	66.46
	R500	39.59	38.98	38.92
	R750	29.60	29.40	29.40
	R1000	23.04	22.95	22.95

C: Capture (Sample I) R: Recapture (Sample II)

In this study, when the first sample size is big, Chapman and Bailey methods gave better estimation results than Lincoln – Petersen method and Bailey method gave a little bit better estimation than Chapman method when the two methods are compared.

The population used for this study N=1953, the size of first and second sample ranged from 1.3% to 51% of the population. In similar study, Yang and Pal (2010) conducted a study on population size N=1000 and the sample sizes were ranging from 1% to 25% of the population.⁹ In this study a larger population N=1953, sample sizes ranging from 1.3% to 51% of the population and more scenarios were considered to assess and compare the performance of the estimators on different sample sizes. Considering the proportion of population and sample sizes of Yang and Pall (2010) and this study, in terms of small and large sample sizes both studies shows similar results.

El Allaki et. al. (2015) evaluated the performance of some estimators in their work on small medium and large population sizes. In their study they showed that Chapman method gave more error than Lincoln – Petersen method in small and large population while in medium size population Lincoln – Petersen and Chapman showed similar performance.²

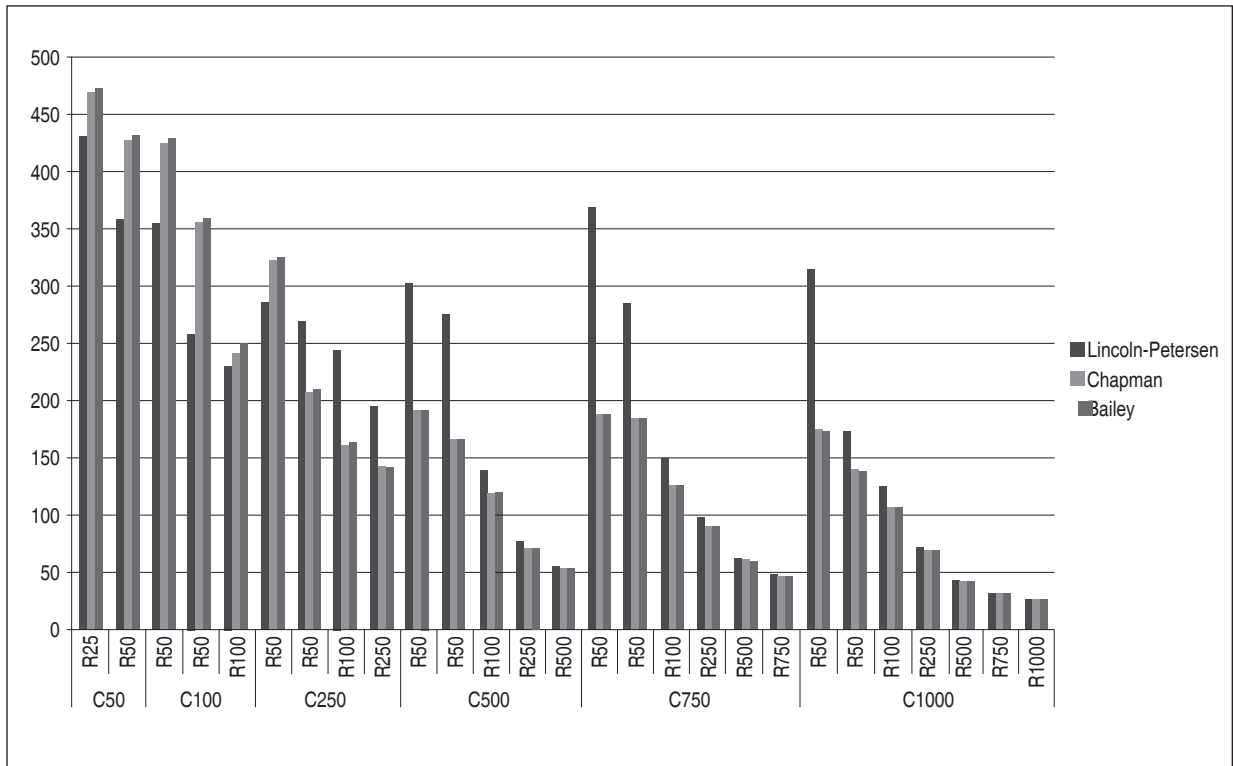


FIGURE 1: Graphical representation of the estimators RMSE on different sample sizes n_1, n_2 .

CONCLUSION

At the conclusion part of this study we remark that, when dealing with small samples Lincoln – Petersen should be considered but when no element recaptured in the second sample researchers should consider Chapman and Bailey methods as alternative. When dealing with big size sample one and small size sample two Chapman and Bailey methods should be considered and when both samples are large all the methods discuss in this study can be considered.

Source of Finance

During this study, no financial or spiritual support was received neither from any pharmaceutical company that has a direct connection with the research subject, nor from a company that provides or produces medical instruments and materials which may negatively affect the evaluation process of this study.

Conflict of Interest

No conflicts of interest between the authors and / or family members of the scientific and medical committee members or members of the potential conflicts of interest, counseling, expertise, working conditions, share holding and similar situations in any firm.

Authorship Contributions

Idea/Concept: Musa Bashir Albışır, İlker Ercan; **Design:** Musa Bashir Albışır, İlker Ercan; **Control/Supervision:** Musa Bashir Albışır, İlker Ercan; **Data Collection and/or Processing:** Musa Bashir Albışır, İlker Ercan; **Analysis and/or Interpretation:** Musa Bashir Albışır, İlker Ercan; **Literature Review:** Musa Bashir Albışır, İlker Ercan; **Writing the Article:** Musa Bashir Albışır, İlker Ercan; **Critical Review:** Musa Bashir Albışır, İlker Ercan; **Materials:** Uludağ Üniversitesi

Informing

Due to the presence of the name of the journal editor’s among the authors, the assessment process of the study was conducted by the guest editor.

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