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A Review of Some Change-Point Detection Methods

Bazı Değişim Noktası Algılama Yöntemlerinin İncelenmesi

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ABSTRACT The irregularity detection problem has an extremely wide range of applications that varies from vital critical to completely scientific. Statistically testing structural change problems is an important subject. In recent years, the change-point analysis method has proven to be a useful analytic tool in analyzing time series datasets and identifying underlying trends. Most of the results presented in previous studies show that change-point analysis is capable of revealing the existence of hidden change points in time series or sequence datasets. The change point problems have been much scrutinized in the statistical literatures. Change point detection (CPD) can be defined as the problem of hypothesis testing between two alternatives. In general the problem concerns detecting whether or not a change has taken place. A lot of change point detection methods are designed, developed and adapted for change point detection. The article firstly presents an overview of the methods commonly used to determine the point of change. The main purpose of this study is to introduce the most popular and uncomplicated three different change point detection methods (Cumulative Sum (CUSUM) method, Pettitt method and Mean Squared Error Minimization method). In the presence of time series data, the characteristics of the exchange point detection tests presented in this article have been investigated. In addition, an example has been solved carried out to how the calculate the change point by the methods considered. As a result, it has come to the conclusion that these methods are simple and effective methods to detect significant changes over time.

Keywords: Time series analysis; sequential data analysis; change point detection; CUSUM; pettitt method; mean squared error minimization

ÖZET Bir veri setinde düzensizlik algılama problemi, yaşam açısından kritik olandan, tamamen bilimsel olana kadar değişen çok geniş bir uygulama alanına sahiptir. İstatistiksel olarak yapısal değişim problemlerini test etmek önemli bir konudur. Son yıllarda, değişim noktası analizi yöntemlerinin, zaman serisi verilerini analiz etmede ve temel eğilimlerin tanımlanmasında faydalı bir analiz aracı olduğu kanıtlanmıştır. Önceki çalışmalarda sunulan sonuçların çoğu değişim noktası analizinin, zaman serilerinde veya sıralı veri kümelerindeki gizli değişim noktalarının varlığını açığa ortaya çıkarabildiğini göstermektedir. İstatistiksel literatürde değişim noktası belirleme problemi çok araştırılmış ve incelenmiştir. Değişim noktası tespiti iki alternatif arasındaki hipotez testi problemi olarak tanımlanabilir. Genel olarak sorun, bir değişikliğin meydana gelip gelmediğinin saptanmasıyla ilgilidir. Pek çok değişim noktası belirleme yöntemi, değişim noktası tespiti için tasarlanmış, geliştirilmiş ve uyarlanmıştır. Makalede öncelikle, yaygın olarak değişim noktası belirleme sorununa uygulanan yöntemlere genel bir bakış sunulmaktadır. Bu çalışmanı temel amacı ise, en popüler ve karmaşık olmayan üç farklı değişim noktası tespit yöntemini (Birikimli Toplam yöntemi, Pettitt yöntemi ve Hata Kareler Ortalaması Minimizasyonu yöntemi) tanıtmaktır. Zaman serileri verilerinin varlığında, bu makalede sunulan değişim noktası algılama testlerinin hem performansları değerlendirilmiş hem de özellikleri araştırılmıştır. Ayrıca, dikkate alınan yöntemler ile değişim noktasını hesaplamak için bir örnek çözülmüştür. Sonuç olarak, bu yöntemlerin, süreçteki önemli değişiklikleri tespit etmek için basit ve etkili yöntemler olduğu sonucuna varılmıştır.

Anahtar Kelimeler: Zaman serileri analizi; ardışık veri analizi; değişim noktası belirleme; CUSUM; pettitt yöntemi; hata kareler ortalaması minimizasyonu

The change-point analysis has been enhanced following the methodological article (Page, 1954), where the cumulative sum control chart was introduced and a technically included branch of mathematical statistics.[1,2] The change point analysis has developed fastly, with multiple change points, different types of data and other assumptions being considered. Change points also appear under a variety of synonyms (like segmentation, structural breaks, break points, regime switching and detecting disorder) across a variety of scientific fields. In a wide range of literature specially including quality control, economics, medicine, environment etc. the concept of change point can be found.[3] In statistics, the problem of change-point detection is consider over the last several decades. If two distributions are significantly different a typical statistical formulation of change-point detection is take into consideration probability distributions from which data in the past and present intervals are generated, and regard the target time point as a change point if two distributions are significantly different.[4] Appropriate statistical tests are necessary to determine whether data need segmentation, where the change-points should be localized and how many divisions to make. Techniques of varying complexity have been evolved for identifying change-points in a time series but as a domain change point analysis is characterized both by its specialized focus and its technical complexity.[5] Change point analysis is one of the statistical approaches widely adopted by researchers to identify change points in a data flow and has shown the potential to detect sudden change candidate points. A change point is a point where a shift in the data pattern occurs. It is defined as a point at which the parameters (such as mean, variance, and trend) of an considered distribution or the parameters of a model used to describe time series suddenly changes.[6] Namely, let $\{Y_t\}$ be an observed process in discrete time. The time moment t^* is a change point with F_1 and F_2 distribution functions if

$$\begin{Bmatrix} Y_t \sim F_1 & t < t^* \\ Y_t \sim F_2 & t \geq t^* \end{Bmatrix}$$

The aim is to find out t^* as soon as possible after its occurence, under some prearranged false-alarm rate α.[7,8] In this way, the detection and estimation of the change points dividing the statistical models into homogeneous segments is a fast developing field in statistical theory and methods. A common feature of all change point detection methodologies is that the problem is very natural, the data are divided in different ways, and the separation between the two parts is to investigate heterogeneity according to some criteria.[9] Time series data are time series measurement sequences that describe the behavior of the systems. These behaviors may change over time due to external events and / or internal systematic changes to the dynamic / distribution. Change point detection (CPD) is the problem of finding instantaneous changes in the data when a characteristic of the time sequence changes. The change points are sudden changes in the time series data. Such sudden changes may represent transitions between states. Detection of change points is useful in modeling and predicting time series and is found in application areas.[10] Several methods (frequently used methods are likelihood ratio procedure, Bayesian solution, cumulative sum method, information criterion and wavelet transform method) have been proposed in the literature to test or estimate change points in mean, variance, and regression parameters. A study of the literature on change point analysis shows that most of the previous studies concentrated on the single point of change in random sequence. However, the issue of multiple change points has not been taken into account by many authors. In order to determine the number of change points and locations in a multi-dimensional random process, a method known as a binary partition procedure and proven to be consistent has been proposed. This binary partitioning procedure has the benefit of simultaneously detecting the number of change points and their locations and recording a lot of computation time.[11] More than one change issue can often be encountered in practice. Suppose a change is already detected. It is then tried to decide whether to examine two parts of the original series separately, that is, the part before the change and the part after the change, and whether these two parts will be accepted as stationary or another change will not be discovered. In the case where another change point is detected, all the change points can proceed in the same way until they are found.[12] Since change point analysis is not developed for a unique data type, it is very flexible in applications, assuming that the data is in chronological order.[13] Change point problem is primarily caused by the quality control process, which is concerned about the output of a production line and which seeks to find any deviation from an acceptable standard of products. The sudden change problem is often compared in various experimental and mathematical sciences. From a statistical point of view, it should be determined whether there is a statistically significant change point in a chronologically ordered sequence of data. If the situation is statistically significant, then the change point should also be estimated.[11] In one way, the classical two-sample and multi-sample problem can be thought of as a special case of the problem of general change point described above. The important difference is that in the classical case, the possible positions of change are known in advance, and in the above formula, the most important question is to determine these possible positions.[14] Change point analysis is an effective tool to understand basic information in different data. In particular, the result of a reasonable change point detection method can be used effectively and predictably; because it provides a key to solving the non-stationary or non-homogeneous problem.[15] According to Page (1955), Change Point Analysis (CPA) was developed to test Shewhart's 3-sigma control chart procedures for testing a change in the parameter that took place at an unknown time to improve quality control. CPA is typically used in ecology, biostatistics, or behavioral statistics. Today, methodology is widely used as a statistical tool to solve change point problems in all areas of science and technology.[16] Change point analysis is used in many areas such as archeology, econometrics, epidemiology, medicine, reliability and financial applications. In many statistical applications it is important to consider that the values of observations can change from an unknown time point. The time when observational values change is called the change point.[17] In many practical applications it is assumed that the structural stability of the statistical models and the fact that this basic assumption must be tested before it can be applied. This is called the analysis of structural breaks or change points that lead to the development of various theoretical and practical results.[18] Did a change occur? Did more than one change occur? When did the changes occur? With what confidence did the changes occur? All these questions and more can be answered by making a change point analysis. Attribute data may be performed on any time-dependent data, including data from nonnormal distributions, badly treated data such as particle counts and complaint data, and data with outliers.[19]

A change-point analysis has numerous advantages:

- It is stronger in detecting smaller continuous changes,

- The detection of multiple changes better characterizes changes such as ensuring relevant confidence levels and establishing confidence intervals for the timing of changes,

- Reduces the number of false detections by controlling the rate of change error. As a result, it is very reliable that every change detected is real,

- It is resistant to outliers and can be further strengthened by performing a change point analysis in its ranks,

- It is more flexible,

- Especially when there are large data sets and numerous changes, their use and interpretation is simpler.

Despite its numerous advantages, change point analysis has two shortcomings. First, CPA can not detect isolated and abnormal points that are critical in explaining the emergence of the underlying mechanisms behind a particular event or process in a process. Second, the bootstrapping approach will not produce the same results every time it is performed.[19] Furthermore, this approach requires completely quantifiable variables and data. CPA is a simple and effective method to detect significant changes in the process, can be compared with each change point (t0,...t + n).[16] Since the mid-twentieth century, the problem of retrospective change point has been extensively addressed in the statistical and engineering literature. Formally, the problem is that of hypothesis testing:

$$H_0 : X_1, X_2, \ldots\ldots, X_n \sim F_1 \quad (1)$$

$$H_1 : X_i \sim F_1,\ X_j \sim F_2, \qquad i = 1, 2, \ldots\ldots, v-1, j = v, v+1, \ldots\ldots, n,$$

where $X_1, X_2, \ldots\ldots, X_n$ is a sample of independent observations, F_1 and F_2 are distribution functions with corresponding density functions f_1 and f_2, respectively. The distribution functions F_1 and F_2 are not necessary known. The unknown parameter $v,\ 2 \leq v \leq n$ is called a change point. According to statistical literature, problem 1 has been investigated in parametric and nonparametric forms depending on the assumptions made on F_1 and F_2 distribution functions. In the parametric case 1, it is assumed that F_1 and F_2 of the distribution functions have known forms, including some unknown parameters. In the nonparametric case of 1, it is assumed that the functions F_1, F_2 are completely unknown.[20] Much of the literature is concerned with the question of structural changes through hypothesis testing, rather than focusing on predicting potential change points. If the null hypothesis of structural change is rejected, the location of the change should be estimated and the statistical analysis should be modified to handle different stochastic properties before and after the change point.[21]

A change-point represents a transition between different states in a process that constitutes the time-series data. In the case of independent observations, change point problems have been extensively researched and parametric and nonparametric tests have been studied. A study of the literature on change point analysis shows that most of the previous studies concentrated on the single point of change in random sequence. Depending on the purpose of the data collected, it is very important to determine the points of change necessary for decision-making or understanding of certain scientific issues. Several methods have been introduced to detect the variance of mean, variance, regression line slope, hazard ratio, or non-parametric distribution for various models.[22] In this study, the three most popular CPA methods to detect one or several change points, CUSUM, Pettitt Method and Mean Squared Error Minimization Method are reviewed. The main aim of this article is to concentrate on the methods considered rather than provide a comprehensive review of the available literature.

CUSUM

The average change in the sequence of normal random variables can be estimated from a cumulative sum test scheme.[23] The task of detecting changes over time also concerns other areas where statistical properties change in time series. A standard approach to the analysis of such temporal data is the cumulative sum method introduced by Page (1954) for the first time. The basic step of this method involves repeated calculations to calculate deviations from a reference state, to calculate the sum of residuals, and to detect changes in the statistical properties.[24] This method is a very simple and realistic method because it perceives the change without assuming any functional form of the time series. A CUSUM procedure is used to draw the accumulated residuals between a measured variable and to test for evidence that residual time series is non-stationary or non-homogeneous. Cumulative sums are not cumulative sums of values. Instead, it is the cumulative sum of the differences between the values and the mean. These differences are summed to zero, so the cumulative sum always ends at zero.[25,26] Let $X_1, X_2, \ldots\ldots, X_n$ represent the n data points. From this, the cumulative sums $S_0, S_1, X_2, \ldots\ldots, X_n$ are calculated. The cumulative sums are calculated as follows:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$S_0 = 0$$

$$S_i = S_{i-1} + (x_i - \bar{x})$$

$$Change\ Point \rightarrow |S_k| = \max_{i=0,1,\ldots,n} |S_i|$$

$|S_k|$ is the point farthest from zero in the CUSUM. The point k estimates last point before the change occured. The point k +1 estimates the first point after the change. However, there are a number of problems in this method, because it requires a significant skill to interpret correctly. To solve this problem, a level of confidence can be used with a bootstrapping approach, possibly to measure a possible change timing and to estimate the uncertainty associated with a change point.[25] After performing bootstrap from observed data, the measured CUSUM difference should be compared with each randomized CUSUM difference $(S_{diff} = \max S_i - \min S_i)$ of results by bootstrap to determine where it lies in the distribution of randomized values. Also, the Confidence Level (*CL*) in CUSUM method of an occurrence of change can be calculated by

$$CL = \frac{R}{N} * 100$$

where, R is the number of randomizations when $S_{diff} > S_{diff}^{bootstrap_i}$ and *N* is the total number of realizations. Due to simplicity, the CUSUM idea has been used in many applications. Interpretation of an obtained result of the CUSUM method requires some practice. For a while, suppose that the values are above the general average. Most of the values added to the cumulative aggregate will be positive and the total will increase continuously. CUSUM with upward slope shows a period when the values are above the general average. Similarly, a downward sloping CUSUM shows a period of time when the values tend to be below the general average. It shows a sudden change in CUSUM direction, a sudden shift or a change in the average. A sudden change in direction indicates that the averages have changed over this time.[26] If (1) the data are independent and uniformly distributed, and (2) the distributions before and after the change are known the CUSUM method has been shown to be asymptotically optimal. Further studies have generalized these results by demonstrating optimality for data that are not necessarily independent and identically distributed.[24] When the basic process is Gaussian, the CUSUM test has good properties. However, the CUSUM test is not robust against possible outliers in the data, because the sum $\sum_{i=1}^{k} x_i$ can vary greatly when there are outliers.[27] CUSUM statistics have been widely adopted for segmenting both univariate and multivariate data. For univariate data segmentation, the CUSUM statistic is calculated over time, and these CUSUM series are typically examined to find a change point as the maximum value in the absolute value is reached. When combined with a binary segmentation algorithm, CUSUM statistics can recursively perceive multiple change points recursively.[28]

PETTITT METHOD

It should be assumed that parametric methods should know the initial distribution functions of random variables that are not satisfied in practice. For this reason, alternative non-parametric methods have been taken into consideration by many researchers.[29,30] Pettitt (1979), proposed a test statistic to determine the mean changes based on Mann-Whitney two sample test. The Pettitt method, a rank-based test method, has been widely used to determine the change point in the mean value of the observed series. Traditionally, the benchmark test is assumed to be free of distribution and not sensitive to outliers and distorted distributions. To achieve the identification of change point, a statistical index $U_{k,n}$ is defined as follows;[31]

$$D_{ij} = sgn(x_i - x_j)$$

$$U_{k,n} = \sum_{i=1}^{k} \sum_{j=k+1}^{n} D_{ij}$$

$$Change\ Point \rightarrow K_n = \max_{1 \le k \le n} |U_{k,n}|$$

The statistic $U_{k,n}$ is equivalent to a Mann-Whitney statistic for testing $X_1, X_2, \ldots\ldots, X_k$ and $X_{k+1}, X_{k+2}, \ldots\ldots, X_n$ come from the same population. The significance probability associated with K_n is approximately evaluated as

$$p \sim 2exp\left(\frac{-6K_n^2}{n^2 + n^3}\right)$$

Given a certain significance level α, if *p*<α, the null hypothesis is rejected and it is concluded that x_k is a significant change point at level α. The approach is widely applied to identify a single point of change in continuous data.

MEAN SQUARED ERROR MINIMIZATION

In a single change point array, it is attempted to explain the sequence optimally using two fixed functions. That is, the most suitable point is to divide the sequence so that the two halves of the sequence are very close to the sample means.[32] Formally,

$$MSE(k) = \sum_{i=1}^{k} (x_i - \bar{x}_1)^2 + \sum_{i=k+1}^{n} (x_i - \bar{x}_2)^2$$

$$\bar{x}_1 = \frac{\sum_{i=1}^{k} x_i}{k}$$

$$\bar{x}_2 = \frac{\sum_{i=k+1}^{n} x_i}{n-k}$$

$$Change\ Point \rightarrow \min_{1 \le k \le n} MSE(k)$$

The Mean Squared Error Minimization method relies on the idea of two divisions, from 1 $to\ k$ and k+1 $to\ n$, to estimate the average of each segment of the data and then see how well the data is to the two predicted averages. The k value that minimizes the $MSE(k)$ is the best estimator of the last point before the change.[19]

MATERIAL AND METHODS

Consider the following artificial data set. The foregoing techniques is illustrated by applying in the example given in Table 1.

The S_i, $U_{i,n}$ and $MSE(i)$ values obtained applying methods are shown in Table 2.

TABLE 1: Sample data set.

i	x_i	i	x_i	i	x_i	i	x_i
1	-1.05	11	0.44	21	2.14	31	1.37
2	0.96	12	0.91	22	1.22	32	1.66
3	1.22	13	-0.02	23	-0.24	33	0.10
4	0.58	14	-1.42	24	1.60	34	0.80
5	-0.98	15	1.26	25	0.72	35	1.29
6	-0.03	16	-1.02	26	-0.12	36	0.49
7	-1.54	17	-0.81	27	0.44	37	-0.07
8	-0.71	18	1.66	28	0.03	38	1.18
9	-0.35	19	1.05	29	0.66	39	3.29
10	0.66	20	0.97	30	0.56	40	1.84

$$Change\ Point \rightarrow k = 17, \quad x_{17} = -0.81$$

$$For\ Change\ Point \rightarrow |S_k| = \max_{i=0.1.\dots\dots n} |S_i| = |S_{17}| = 10.715$$

$$For\ Change\ Point \rightarrow K_n = \max_{1\leq k\leq n} |U_{k.n}| = \max_{1\leq k\leq 40} |U_{17.40}| = 232$$

$$p \sim 2exp\left(\frac{-6K_n^2}{n^2+n^3}\right) = 2exp\left(\frac{-6*232^2}{40^2+40^3}\right) = 0.0146$$

$$For\ Change\ Point \rightarrow \min_{1\leq k\leq n} MSE(k) = \min_{1\leq k\leq 40} MSE(17) = 29.823$$

RESULTS

There are a lot of methods for detecting change points in time series and sequence data. In the case of independent observations, change point problems have been extensively researched and parametric and nonparametric tests have been studied. There are also many studies for weakly dependent observations.[33] When the problem of testing the hypotheses given in Equation (1) is stated nonparametrically, it is suggested that the common components of change point detection policies are based on signs and / or ranks. 18 In the literature, there is no comparative study involving the change point detection methods considered. There are studies in which the methods considered in the study are compared with other methods, even in a small number. The likelihood ratio test was slightly less powerful than the cumulative sum test in the middle of the array, but the likelihood ratio test was concluded to be stronger near the extremities.[34] In the study conducted by Lee and Kim (2016), CUSUM method, Bayesian Change Point (BCP) method, and segmentation method by Dynamic Programming (DP) were taken into consideration. According to the results obtained, BCP and DP were slightly superior to CUSUM because their type I errors were lower than that of CUSUM in homogeneous synthetic series.[15] It is recommended that one could use Pettitt method whenever detecting shift, if it exists, at change point close to $n/2$ is most important.[35] Allen et al. (2017) discusses various approaches to change point detection. These are: Mean Squared Error Minimization (MSE), Forcasting Methods, Hidden Markov Models and Maximum Likelihood Estimation. They concluded that the performance of the classifiers was a significant influence on the accuracy of the results of the detection of change points. According to the results obtained for different situations, MSE has the highest sensitivity values compared to other methods.[32]

TABLE 2: Occurrence of potential abrupt shifts.				
i	x_i	S_i	$U_{i,n}$	$MSE(i)$
1	-1.05	-1.569	-35	39,044
2	0.96	-1.127	-24	40,899
3	1.22	-0.426	-4	41,502
4	0.58	-0.364	-5	41,531
5	-0.98	-1.863	-36	40,775
6	-0.03	-2.411	-53	40,428
7	-1.54	-4.470	-92	38,109
8	-0.71	-5.698	-119	36,495
9	-0.35	-6.567	-144	35,386
10	0.66	-6.425	-142	36,064
11	0.44	-6.504	-150	36,264
12	0.91	-6.112	-141	37,121
13	-0.02	-6.651	-156	36,527
14	-1.42	-8.589	-193	33,461
15	1.26	-7.848	-170	34,999
16	-1.02	-9.386	-203	32,391
17	-0.81	-10.715	-232	29,823
18	1.66	-9.573	-200	32,311
19	1.05	-9.042	-185	33,372
20	0.97	-8.590	-172	34,189
21	2.14	-6.969	-135	36,700
22	1.22	-6.267	-115	37,601
23	-0.24	-7.026	-138	36,518
24	1.60	-5.944	-109	37,887
25	0.72	-5.743	-104	38,050
26	-0.12	-6.381	-125	37,093
27	0.44	-6.460	-133	36,813
28	0.03	-6.948	-146	35,821
29	0.66	-6.807	-144	35,759
30	0.56	-6.765	-147	35,466
31	1.37	-5.914	-120	36,554
32	1.66	-4.772	-88	38,010
33	0.10	-5.191	-99	36,903
34	0.80	-4.909	-92	36,843
35	1.29	-4.138	-67	37,655
36	0.49	-4.166	-72	36,747
37	-0.07	-4.755	-91	33,422
38	1.18	-4.093	-74	32,751
39	3.29	-1.322	-35	39,777
40	1.84	0.000	0	41,568

CONCLUSION

In many applications, it is useful to know when structural change occurs. Treating the date of structural change -the breakdate- as an unknown parameter, the issues are how to estimate the breakdate and how to obtain confidence intervals for the breakdate.[36] Assuming the existence of an change point, it is interesting to estimate the exchange point and establish the trust zones. Change point analysis for time series is an increasingly important aspect of statistics. Simply put, a change point is one in which the statistical properties before and after this time point are different. Change point analysis is crucial in both point and theoretical statistics, with many statistical methods that accept naturally occurring potential changes and "no change" setup in the data.[3]

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Conflict of Interest

No conflicts of interest between the authors and / or family members of the scientific and medical committee members or members of the potential conflicts of interest, counseling, expertise, working conditions, share holding and similar situations in any firm.

Authorship Contributions

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