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Power Shanker Distribution and its Application

Power Shanker Dağılımı ve Uygulaması

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ABSTRACT A two-parameter power Shanker distribution, of which Shanker distribution introduced by Shanker (2015) is a particular case, has been proposed and its important statistical properties including shapes of the density and hazard rate function, moments, skewness and kurtosis measures have been discussed. The stochastic ordering of the distribution has been explained. The maximum likelihood estimation has been discussed for estimating its parameters. Finally, applications and goodness of fit of the proposed distribution has been discussed with a real lifetime data and the fit has been compared with power Lindley distribution and one parameter Shanker, Lindley and exponential distributions.

Keywords: Shanker distribution; hazard rate function; moments; stochastic ordering; maximum likelihood estimation; goodness of fit

ÖZET İki parametreli bir güç Shanker dağılımı olan Shanker dağılımı Shanker (2015) tarafından getirilen özel bir durumdur ve yoğunluk şekilleri ve hazard hızı fonksiyonu, momentler, çarpıklık ve basıklık ölçüleri gibi önemli istatiksel özellikleri tartışılmıştır. Dağılımın stokastik sıralaması açıklanmıştır. En çok olabilirlik tahmini parametrelerini tahmin etmek için tartışılmıştır. Son olarak, önerilen dağılımın uygulamaları ve uyum iyiliği, gerçek yaşam boyu verilerle tartışılmış ve uyum, Lindley dağılımı ve bir parametreli Shanker, Lindley ve üstel dağılımlarla karşılaştırılmıştır.

Anahtar Kelimeler: Shanker dağılımı; hazart hızı fonksiyonu; momentler; stokastik sıralama; en çok olabilirlik tahmini; uyum iyiliği

Shanker (2015) introduced a lifetime distribution named Shanker distribution having probability density function (pdf).[1]

$$f_1(y;\theta)=\frac{\theta^2}{\theta^2+1}(\theta+y)e^{-\theta y} \quad ; y>0,\ \theta>0 \tag{1.1}$$

$$= p\,g_1(y;\theta)+(1-p)\,g_2(y;2,\theta)$$

where

$$p=\frac{\theta^2}{\theta^2+1}$$

$$g_1(y;\theta)=\theta e^{-\theta y} \ ; y>0$$

$$g_2(y;\theta)=\frac{\theta^2}{\Gamma(2)}e^{-\theta y}y^{2-1}; y>0$$

The pdf in (1.1) shows that the Shanker distribution is a two – component mixture of an exponential distribution (with scale parameter θ) and a gamma distribution (with shape parameter 2 and scale parameter θ), with mixing proportion $p=\frac{\theta^2}{\theta^2+1}$. That is, Shanker distribution is a convex combination of exponential(θ) and gamma$(2,\theta)$ distributions.

The corresponding cumulative distribution function (cdf) of (1.1) is given by

$$F_1(y;\theta)=1-\left[1+\frac{\theta y}{\theta^2+1}\right]e^{-\theta y}\,;y>0,\theta>0 \tag{1.2}$$

The first four moments about origin of Shanker distribution are obtained as

$$\mu_1'=\frac{\theta^2+2}{\theta(\theta^2+1)},\quad \mu_2'=\frac{2(\theta^2+3)}{\theta^2(\theta^2+1)},\quad \mu_3'=\frac{6(\theta^2+4)}{\theta^3(\theta^2+1)},\quad \mu_4'=\frac{24(\theta^2+5)}{\theta^4(\theta^2+1)} \tag{1.3}$$

The variance of the Shanker distribution can be obtained as

$$\mu_2=\frac{\theta^4+4\theta^2+2}{\theta^2(\theta^2+1)^2} \tag{1.4}$$

Shanker (2015) has discussed some of its mathematical and statistical properties including its shapes for varying values of parameter, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic ordering, mean deviations, order statistics, Bonferroni and Lorenz curves, Renyi entropy measure, stress-strength reliability.[1] Further, Shanker (2015) discussed the estimation of its parameter using both the maximum likelihood estimation and the method of moments along with the applications of the distribution for modeling lifetime data from engineering and medical science.[1] Shanker (2016) obtained the discrete Poisson-Shanker distribution, a Poisson mixture of Shanker distribution, and studied its various mathematical and statistical properties and applications for count data from various fields of knowledge.[2] Shanker (2017 a, 2017 b) has also obtained size-biased and zero-truncated forms of Poisson-Shanker distribution and discussed their statistical properties and applications for modeling count data excluding zero counts from biological sciences.[3,4] Shanker and Hagos (2016) has comparative study on modeling of lifetime data using one parameter Akash, Shanker, Lindley and exponential distributions and observed that there are some lifetime data where these distributions do not give satisfactory fit due to theoretical or applied point of view.[5] The reasons for not giving satisfactory fit are that these one parameter distributions are monotonically decreasing and do not have enough flexibility to model lifetime data of various nature.

It should be noted that the pdf the cdf of Power Lindley distribution (PLD) obtained by Ghitany *et al* (2013) are given by[6]

$$f_2(x;\theta,\alpha)=\frac{\alpha\theta^2}{(\theta+1)}x^{\alpha-1}\left(1+x^{\alpha}\right)e^{-\theta x^{\alpha}}\quad;x>0,\theta>0,\alpha>0 \tag{1.5}$$

$$F_2(x;\theta,\alpha)=1-\left[1+\frac{\theta x^{\alpha}}{\theta+1}\right]e^{-\theta x^{\alpha}}\,;x>0,\theta>0,\alpha>0 \tag{1.6}$$

Note that the PLD is a convex combination of Weibull (α,θ) and a generalized gamma $(2,\alpha,\theta)$ distribution with mixing proportion $\frac{\theta}{\theta+1}$. Ghitany *et al* (2013) has discussed the properties of PLD including the shapes of the density and hazard rate functions, moments, skewness and kurtosis measures, some among others.[6] Further, Ghitany *et al* (2013) has discussed the estimation of parameters of PLD using maximum likelihood estimation and the application of PLD to a real lifetime data.[6] Recall that at $\alpha=1$, the PLD (2.4) reduces to Lindley distribution introduced by Lindley (1958) having pdf and cdf.[7]

$$f_3(x;\theta)=\frac{\theta^2}{(\theta+1)}(1+x)e^{-\theta x}\ ;x>0,\theta>0 \tag{1.7}$$

$$F_3(x;\theta)=1-\left[1+\frac{\theta x}{\theta+1}\right]e^{-\theta x};x>0,\theta>0 \tag{1.8}$$

It should be noted that the Lindley distribution is a convex combination of exponential (θ) and gamma $(2,\theta)$ distributions with their mixing proportion $\frac{\theta}{\theta+1}$. The PLD is a power transformation of Lindley distribution and having two parameters gives better fit than the one parameter Lindley distribution for modeling lifetime data of various nature because the additional parameter has enough flexibility to have different shapes. Lindley distribution has been studied in detail by Ghitany et al (2008).[8]. Lindley distribution has been extended, generalized and modified by many researchers including Zakerzadeh and Dolati (2009), Nadarajah *et al* (2011), Deniz and Ojeda (2011), Bakouch et al (2012), Shanker and Mishra (2013 a, 2013 b), Shanker and Amanuel (2013), Shanker *et al* (2013), are some among others.[9-16]

The main motivation of considering Power Shanker distribution (PSD) are (1) power Lindley distribution, a power transformation of Lindley distribution gives better fit than the Lindley distribution (2) the Shanker distribution gives better fit than the Lindley distribution and the Power Shanker distribution (PSD), a power transformation of Shanker distribution having two parameters is expected to give better fit than both Lindley and PLD. Keeping these two reasons in mind it is of interest and expected that PSD will give better fit than Shanker distribution, Lindley distribution and PLD.

In this paper an attempt has been made to obtain power Shanker distribution (PSD) which includes Shanker distribution as a particular case. The natures of its pdf, cdf and hazard rate functions have been discussed graphically. Moments and moments based measures including coefficients of skewness and kurtosis has been obtained. The stochastic ordering of PSD has been discussed. The maximum likelihood estimation has been discussed for estimating its parameters. The goodness of fit of PSD has been shown by an example and the fit has been compared with one parameter exponential, Lindley and Shanker distributions and two-parameter Power Lindley distribution (PLD).

POWER SHANKER DISTRIBUTION

Assuming the power transformation $X=Y^{1/\alpha}$ in (1.1), the pdf of the random variable X can be obtained as

$$f_4(x;\theta,\alpha)=\frac{\alpha\theta^2}{(\theta^2+1)}x^{\alpha-1}\left(\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}}\ ;x>0,\theta>0,\alpha>0 \tag{2.1}$$

$$=p\,g_3(x;\theta,\alpha)+(1-p)\,g_4(x;\theta,\alpha) \tag{2.2}$$

where

$$p = \frac{\theta^2}{\theta^2 + 1}$$

$$g_3(x;\theta,\alpha) = \alpha\,\theta\,x^{\alpha-1}e^{-\theta x^{\alpha}} \;\; ; x > 0$$

$$g_4(x;\theta,\alpha) = \alpha\,\theta^2 x^{2\alpha-1}e^{-\theta x^{\alpha}} ; x > 0$$

We would call the density in (2.1) a power Shanker distribution (PSD). It can be easily shown that PSD is a two- component mixture of Weibull distribution (with shape parameter $\alpha$and scale parameter θ), and a generalized gamma distribution (with shape parameters 2, α and scale parameterθ) with mixing proportion $p = \frac{\theta^2}{\theta^2 + 1}$. Like Lindley distribution, Shanker distribution and PLD, PSD is also a convex combination of Weibull(α,θ) and generalized gamma $(2,\alpha,\theta)$ distributions with mixing proportion $p = \frac{\theta^2}{\theta^2 + 1}$, which is different from the mixing proportion of PLD. The corresponding cumulative distribution function of PSD (2.1) can be obtained as

$$F_4(x;\theta,\alpha) = 1 - \left[1 + \frac{\theta x^{\alpha}}{\theta^2 + 1}\right]e^{-\theta x^{\alpha}} ; x > 0, \theta > 0, \alpha > 0 \tag{2.3}$$

The nature of the pdf of PSD for varying values of parameters has been shown in Figure 1. It is obvious from the pdf plot of PSD that it takes different shapes including monotonically decreasing, positively skewed, negatively skewed, and symmetrical with platykurtic, mesokurtic and leptokurtic curves for varying values of parameters. This means that it is applicable for modeling data of various natures. The nature of the cdf of PSD for varying values of parameters has been shown in Figure 2.

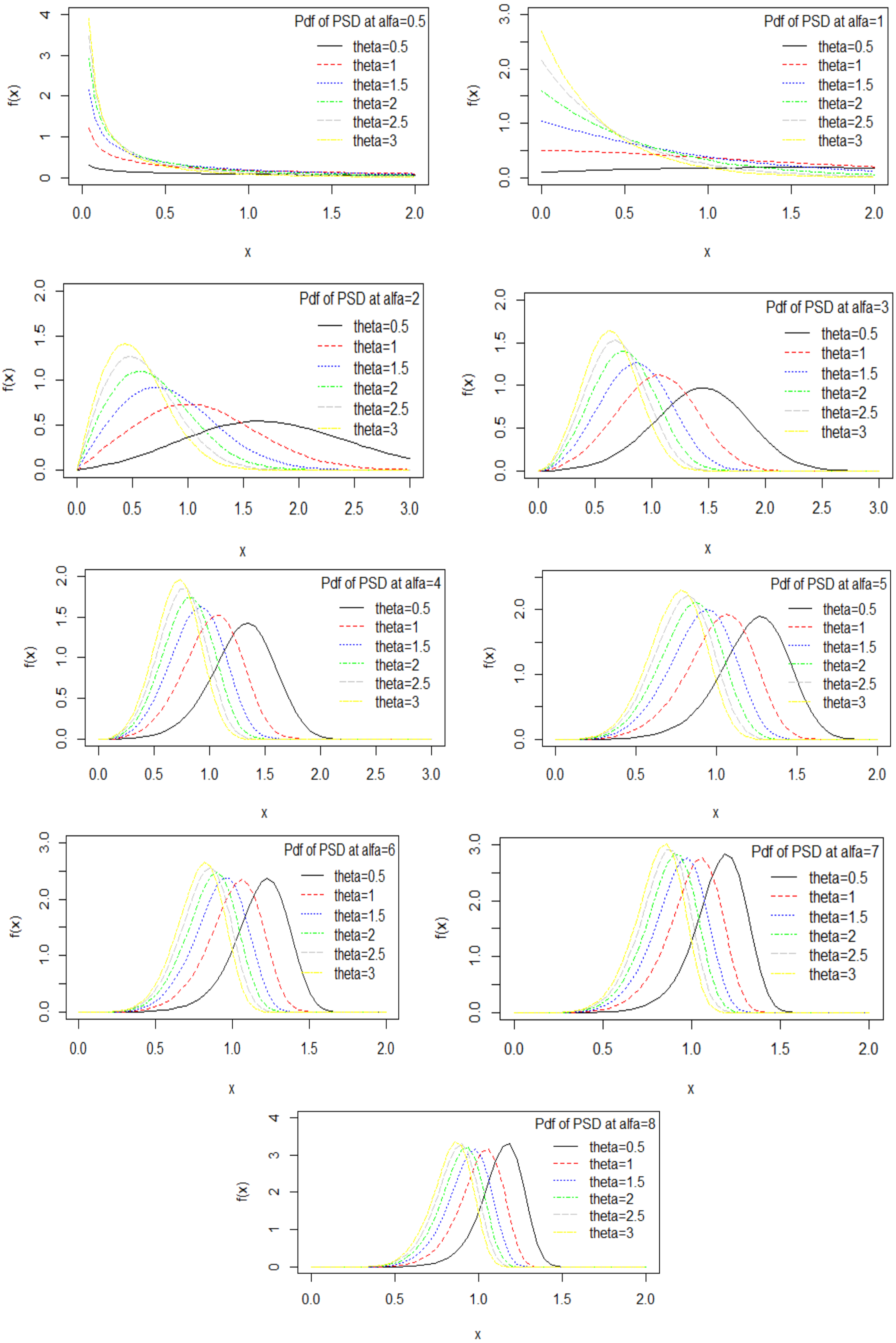

FIGURE 1:pdf plots of PSD for varying values of parameters. θ and α.

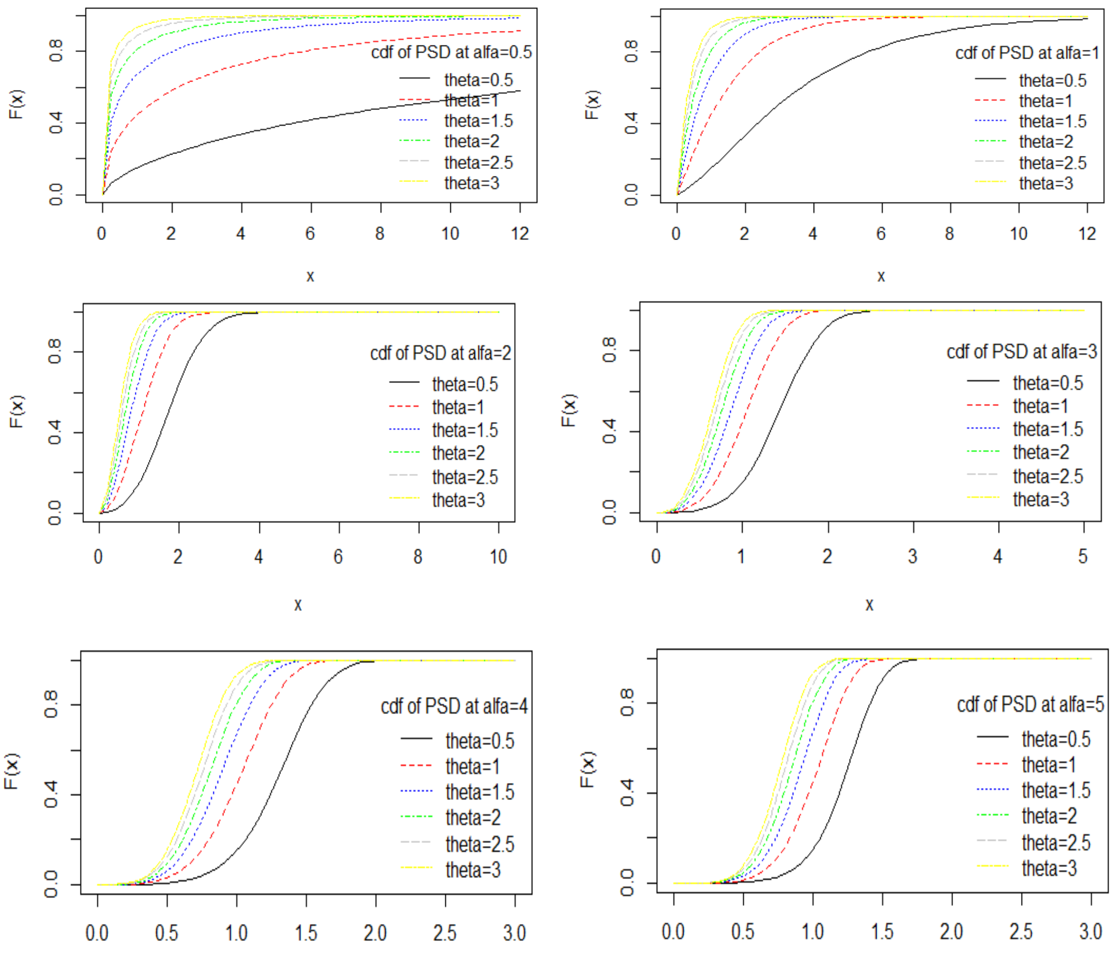

FIGURE 2: cdf plots of PSD for varying values of parameters θ and α.

SURVIVAL AND HAZARD RATE FUNCTIONS

The survival function, $S(x)$ and hazard rate function, $h(x)$ of PSD can be obtained a

$$S(x)=1-F(x;\theta,\alpha)=\left[\frac{(\theta^2+1)+\theta x^{\alpha}}{\theta^2+1}\right]e^{-\theta x^{\alpha}}\;;x>0,\theta>0,\alpha>0$$

$$h(x)=\frac{f(x)}{S(x)}=\frac{\alpha\theta^2(\theta+x^{\alpha})x^{\alpha-1}}{(\theta^2+1)+\theta x^{\alpha}};x>0,\theta>0,\alpha>0 \tag{3.1}$$

The behavior of $h(x)$ at $x=0$ and $x=\infty$, respectively, are given by

$$h(0)=\begin{cases}\infty & \text{if } 0<\alpha<1\\ \dfrac{\theta^3}{\theta^2+1} & \text{if } \alpha=1\\ 0 & \text{if } \alpha>1\end{cases}\text{, and}\quad h(\infty)=\begin{cases}0 & \text{if } \alpha<1\\ \theta^2 & \text{if } \alpha=1\\ \infty & \text{if } \alpha>1\end{cases}$$

The nature of hazard rate function of PSD has been shown graphically for varying values of its parameters and presented in Figure 3.

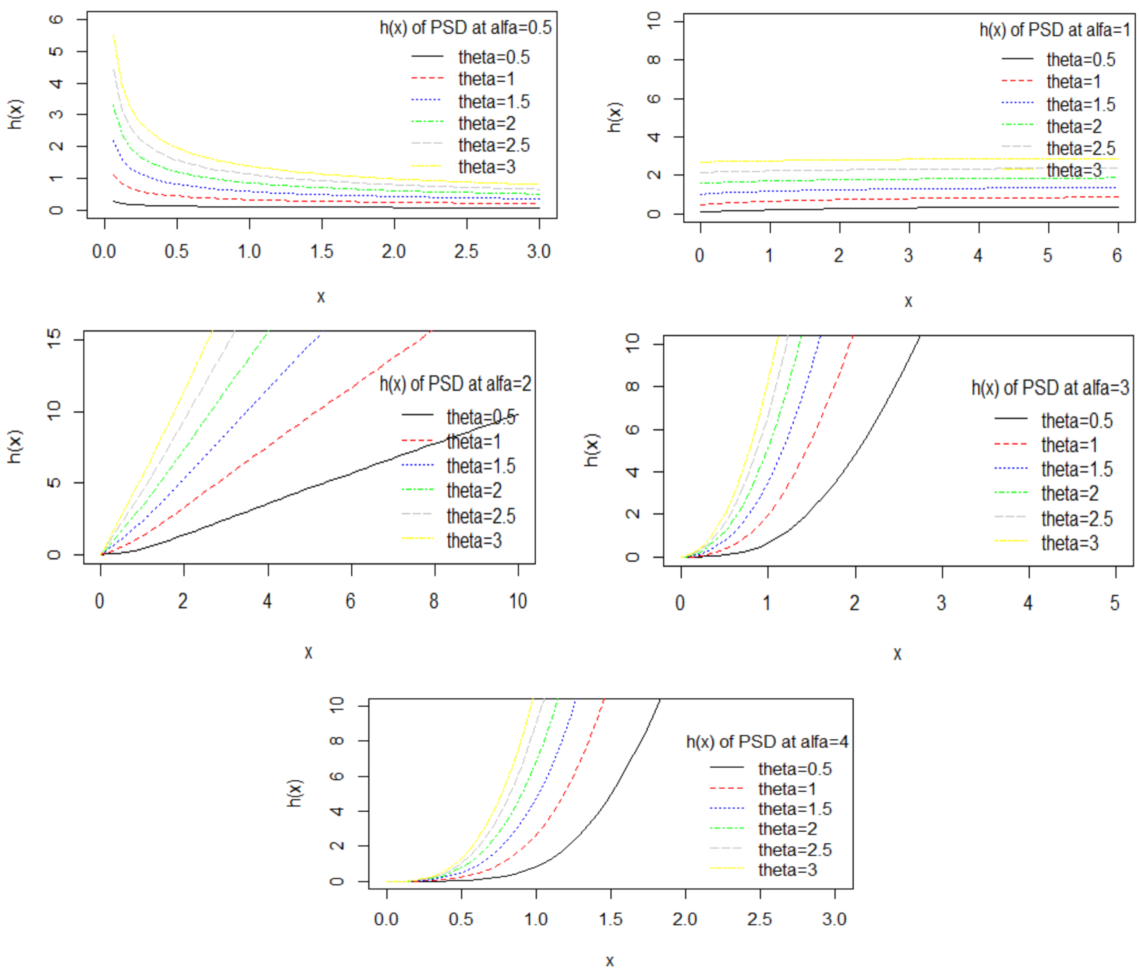

FIGURE 3: Hazard rate function of PSD for varying values of parameters θ and α.

MOMENTS AND RELATED MEASURES

Using the mixture representation (2.2), the r th moment about origin of PSD can be obtained as

$$\mu_r' = E\left(X^r\right) = p\int_0^\infty x^r g_3\left(x;\theta,\alpha\right)dx + \left(1-p\right)\int_0^\infty x^r g_4\left(x;\theta,\alpha\right)dx$$

$$= \frac{r\Gamma\left(\frac{r}{\alpha}\right)\left(\theta^2+r+\alpha\right)}{\alpha^2\theta^{r/\alpha}\left(\theta^2+1\right)};r=1,2,3,.... \qquad (4.1)$$

It should be noted that at $\alpha=1$, the above expression will reduce to the r th moment about origin of Shanker distribution (1.1) and is given by

$$\mu_r' = \frac{r!\left(\theta^2+r+\alpha\right)}{\theta^r\left(\theta^2+1\right)};r=1,2,3,...$$

Therefore, the mean and the variance of the PSD, respectively, are obtained as

$$\mu = \frac{\Gamma\left(\frac{1}{\alpha}\right)\left(\theta^2+\alpha+1\right)}{\alpha^2\theta^{1/\alpha}\left(\theta^2+1\right)}$$

$$\sigma^2 = \frac{2\Gamma\left(\frac{2}{\alpha}\right)\left(\theta^2+\alpha+2\right)\alpha^2\left(\theta^2+1\right)-\left(\Gamma\left(\frac{1}{\alpha}\right)\right)^2\left(\theta^2+\alpha+1\right)^2}{\alpha^4\theta^{2/\alpha}\left(\theta^2+1\right)^2}.$$

The mean and the variance of PSD reduce to the corresponding mean and the variance of Shanker distribution given in (1.3) and (1.4) at $\alpha=1$. The skewness and kurtosis measures of PSD, upon substituting for the raw moments, can be obtained using the expressions

$$\text{Skewness} = \frac{\mu_3' - 3\mu_2' + 2\mu^3}{\sigma^3} \quad \text{and Kurtosis} = \frac{\mu_4' - 4\mu_3'\mu + 6\mu_2'\mu^2 - 3\mu^4}{\sigma^4}.$$

STOCHASTIC ORDERING

Stochastic ordering of positive continuous random variables is an important tool for judging their comparative behavior. A random variable X is said to be smaller than a random variable Y in the

(i) stochastic order $(X \le_{st} Y)$ if $F_X(x) \ge F_Y(x)$ for all x

(ii) hazard rate order $(X \le_{hr} Y)$ if $h_X(x) \ge h_Y(x)$ for all x

(iii) mean residual life order $(X \le_{mrl} Y)$ if $m_X(x) \le m_Y(x)$ for all x

(iv) likelihood ratio order $(X \le_{lr} Y)$ if $\frac{f_X(x)}{f_Y(x)}$ decreases in x.

The following important interrelationships due to Shaked and Shanthikumar (1994) are well known for establishing stochastic ordering of distributions.[17]

$$\begin{array}{l} X \le_{lr} Y \Rightarrow X \le_{hr} Y \Rightarrow X \le_{mrl} Y \\ \qquad\quad\;\; \Downarrow \\ \qquad\;\; X \le_{st} Y \end{array}$$

The PSD is ordered with respect to the strongest 'likelihood ratio' ordering as established in the following theorem:

Theorem: Let $X \sim \text{PSD}(\theta_1,\alpha_1)$ and $Y \sim \text{PSD}(\theta_2,\alpha_2)$. If $\theta_1 > \theta_2$ and $\alpha_1=\alpha_2$ (or $\alpha_1<\alpha_2$ and $\theta_1=\theta_2$) then $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$.

Proof: We have

$$\frac{f_X(x)}{f_Y(x)} = \frac{\alpha_1\theta_1^2\left(\theta_2^2+1\right)}{\alpha_2\theta_2^2\left(\theta_1^2+1\right)} x^{\alpha_1-\alpha_2}\left(\frac{\theta_1+x^{\alpha_1}}{\theta_2+x^{\alpha_2}}\right)e^{-\left(\theta_1 x^{\alpha_1}-\theta_2 x^{\alpha_2}\right)} \quad ; \; x>0$$

Now

$$\ln\frac{f_X(x)}{f_Y(x)} = \ln\left[\frac{\alpha_1\theta_1^2\left(\theta_2^2+1\right)}{\alpha_2\theta_2^2\left(\theta_1^2+1\right)}\right]+\left(\alpha_1-\alpha_2\right)\ln x+\ln\left[\frac{\theta_1+x^{\alpha_1}}{\theta_2+x^{\alpha_2}}\right]-\left(\theta_1 x^{\alpha_1}-\theta_2 x^{\alpha_2}\right)$$

This gives

$$\frac{d}{dx}\ln\frac{f_X(x)}{f_Y(x)}=\frac{\alpha_1-\alpha_2}{x}+\frac{\left(\alpha_1\theta_2x^{\alpha_1-1}-\alpha_2\theta_1x^{\alpha_2-1}\right)+\left(\alpha_1x^{\alpha_1+\alpha_2-1}-\alpha_2x^{\alpha_1+\alpha_2-1}\right)}{\left(\theta_1+x^{\alpha_1}\right)\left(\theta_2+x^{\alpha_2}\right)}$$
$$-\left(\alpha_1\theta_1x^{\alpha_1-1}-\alpha_2\theta_2x^{\alpha_2-1}\right)$$

Thus, for $\theta_1>\theta_2$, and $\alpha_1=\alpha_2$ (or $\alpha_1<\alpha_2$ and $\theta_1=\theta_2$), $\frac{d}{dx}\ln\frac{f_X(x)}{f_Y(x)}<0$. This means that $X\leq_{lr}Y$ and hence $X\leq_{hr}Y$, $X\leq_{mrl}Y$ and $X\leq_{st}Y$.

MAXIMUM LIKELIHOOD ESTIMATES OF PARAMETERS

Let $\left(x_1,x_2,x_3,\ldots,x_n\right)$ be a random sample of size n from PSD (θ,α). Then natural log-likelihood function is given by

$$\ln L=\sum_{i=1}^{n}\ln f\left(x_i\right)$$
$$=n\left[2\ln\theta+\ln\alpha-\ln\left(\theta^2+1\right)\right]+(\alpha-1)\sum_{i=1}^{n}\ln\left(x_i\right)+\sum_{i=1}^{n}\ln\left(\theta+x_i^{\alpha}\right)-\theta\sum_{i=1}^{n}x_i^{\alpha}\cdot$$

The maximum likelihood estimates (MLEs) $\left(\hat{\theta},\hat{\alpha}\right)$ of (θ,α) of PSD (2.1) are the solutions of the following equations

$$\frac{\partial\ln L}{\partial\theta}=\frac{2n}{\theta}-\frac{2n\theta}{\theta^2+1}+\sum_{i=1}^{n}\frac{1}{\left(\theta+x_i^{\alpha}\right)}-\sum_{i=1}^{n}x_i^{\alpha}=0$$

$$\frac{\partial\ln L}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\ln\left(x_i\right)+\sum_{x=1}^{n}\frac{x_i^{\alpha}\ln\left(x_i\right)}{\left(\theta+x_i^{\alpha}\right)}-\theta\sum_{i=1}^{n}x_i^{\alpha}\ln\left(x_i\right)=0$$

These two likelihood equations do not seem to be solved directly because these cannot be expressed in closed form. However, Fisher's scoring method can be applied to solve these equations iteratively. For, we have

$$\frac{\partial^2\ln L}{\partial\theta^2}=-\frac{2n}{\theta^2}+\frac{2n\left(\theta^2-1\right)}{\left(\theta^2+1\right)^2}-\sum_{i=1}^{n}\frac{1}{\left(\theta+x_i^{\alpha}\right)^2}$$

$$\frac{\partial^2\ln L}{\partial\alpha^2}=-\frac{n}{\alpha^2}+\theta\sum_{i=1}^{n}\frac{x_i^{\alpha}\left(\ln\left(x_i\right)\right)^2}{\left(\theta+x_i^{\alpha}\right)^2}-\theta\sum_{i=1}^{n}x_i^{\alpha}\left(\ln\left(x_i\right)\right)^2$$

$$\frac{\partial^2\log L}{\partial\theta\partial\alpha}=-\sum_{i=1}^{n}\frac{x_i^{\alpha}\ln\left(x_i\right)}{\left(\theta+x_i^{\alpha}\right)^2}-\sum_{i=1}^{n}x_i^{\alpha}\left(\ln\left(x_i\right)\right)$$

The MLEs $\left(\hat{\theta},\hat{\alpha}\right)$ of (θ,α) of PSD (2.1) are the solution of the following equations

$$\begin{bmatrix} \frac{\partial^2 \ln L}{\partial \theta^2} & \frac{\partial^2 \ln L}{\partial \theta \partial \alpha} \\ \frac{\partial^2 \ln L}{\partial \theta \partial \alpha} & \frac{\partial^2 \ln L}{\partial \alpha^2} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}} \begin{bmatrix} \hat{\theta}-\theta_0 \\ \hat{\alpha}-\alpha_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial \ln L}{\partial \theta} \\ \frac{\partial \ln L}{\partial \alpha} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}}$$

These equations can be expressed in iterative form as

$$\begin{bmatrix} \hat{\theta} \\ \hat{\alpha} \end{bmatrix} = \begin{bmatrix} \theta_0 \\ \alpha_0 \end{bmatrix} + \begin{bmatrix} \frac{\partial^2 \ln L}{\partial \theta^2} & \frac{\partial^2 \ln L}{\partial \theta \partial \alpha} \\ \frac{\partial^2 \ln L}{\partial \theta \partial \alpha} & \frac{\partial^2 \ln L}{\partial \alpha^2} \end{bmatrix}^{-1}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}} \begin{bmatrix} \frac{\partial \ln L}{\partial \theta} \\ \frac{\partial \ln L}{\partial \alpha} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}}$$

where θ_0 and α_0 are initial values of θ *and* α. These equations are solved iteratively till sufficiently close estimates (up to fifth decimal places) of $\hat{\theta}$ and $\hat{\alpha}$ are obtained. In this paper, R-software has been used to solve above equations to estimate the parameters θ *and* α for the considered example. It should be noted that here quadratic convergence criteria has been used to get the estimates of the parameters.

A NUMERICAL EXAMPLE

In this section, the goodness of fit of PSD using maximum likelihood estimates of parameters to a real data set has been discussed and the fit has been compared with one parameter exponential, Lindley and Shanker distributions and two-parameter PLD. The following data set has been considered for the goodness of fit of the proposed distribution.

The data set represents remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee and Wang (2003).[18]

0.08	2.09	3.48	4.87	6.94	8.66	13.11	23.63	0.20	2.23	3.52	4.98
6.97	9.02	13.29	0.40	2.26	3.57	5.06	7.09	9.22	13.80	25.74	0.50
2.46	3.64	5.09	7.26	9.47	14.24	25.82	0.51	2.54	3.70	5.17	7.28
9.74	14.76	6.31	0.81	2.62	3.82	5.32	7.32	10.06	14.77	32.15	2.64
3.88	5.32	7.39	10.34	14.83	34.26	0.90	2.69	4.18	5.34	7.59	10.66
15.96	36.66	1.05	2.69	4.23	5.41	7.62	10.75	16.62	43.01	1.19	2.75
4.26	5.41	7.63	17.12	46.12	1.26	2.83	4.33	5.49	7.66	11.25	17.14
79.05	1.35	2.87	5.62	7.87	11.64	17.36	1.40	3.02	4.34	5.71	7.93
11.79	18.10	1.46	4.40	5.85	8.26	11.98	19.13	1.76	3.25	4.50	6.25
8.37	12.02	2.02	3.31	4.51	6.54	8.53	12.03	20.28	2.02	3.36	6.76
12.07	21.73	2.07	3.36	6.93	8.65	12.63	22.69				

In order to compare the goodness of fit of the considered distributions values of $-2\ln L$, AIC (Akaike Information Criterion) and K-S Statistic (Kolmogorov-Smirnov Statistic) for the real data set have been computed using maximum likelihood estimates and presented in Table 1. The formulae for computing AIC and K-S Statistic are as follows:

$AIC = -2\ln L + 2k$ and $\text{K-S} = \sup_x |F_n(x) - F_0(x)|$, where k = the number of parameters, n = the sample size, $F_n(x)$ is the empirical (sample) cumulative distribution function and $F_0(x)$ is the theoretical cumulative distribution function. The best distribution is the distribution corresponding to lower values of $-2\ln L$, AIC, and K-S statistic.

TABLE 1: MLE's, -2ln L, AIC, and K-S Statistic of the fitted distributions of data set.

Model	ML Estimates	Standard Error	-2ln L	AIC	K-S Statistic
PSD	$\hat{\theta}=0.35215$ $\hat{\alpha}=0.78821$	0.03914 0.04318	819.83	823.83	0.878
PLD	$\hat{\theta}=0.29486$ $\hat{\alpha}=0.83487$	0.03698 0.04731	822.11	826.11	0.887
Shanker	$\hat{\theta}=0.21412$	0.01308	841.68	843.68	0.908
Lindley	$\hat{\theta}=0.19914$	0.01253	833.79	835.79	0.901
Exponential	$\hat{\theta}=0.10858$	0.00959	824.38	826.38	0.868

It is obvious from the goodness of fit of two-parameter PSD and PLD, and one parameter exponential, Lindley and Shanker distributions in Table 1 that PSD gives better fit and hence it can be considered as an important model for modeling lifetime data from biomedical science and engineering.

The variance-covariance matrix and the 95% confidence intervals (CI) for parameters θ and α of PSD are presented in Table 2.

TABLE 2: Variance-covariance matrix and the 95% confidence intervals for parameters θ and α of PSD.

Parameters	Variance-Covariance Matrix		95% CI	
	$\hat{\theta}$	$\hat{\alpha}$	Lower	Upper
$\hat{\theta}$	0.001532	- 0.001426	0.280818	0.434425
$\hat{\alpha}$	-0.001426	0.001865	0.705159	0.874550

The profile of the maximum likelihood estimates of parameters θ and α of PSD has been shown graphically in Figure 4.

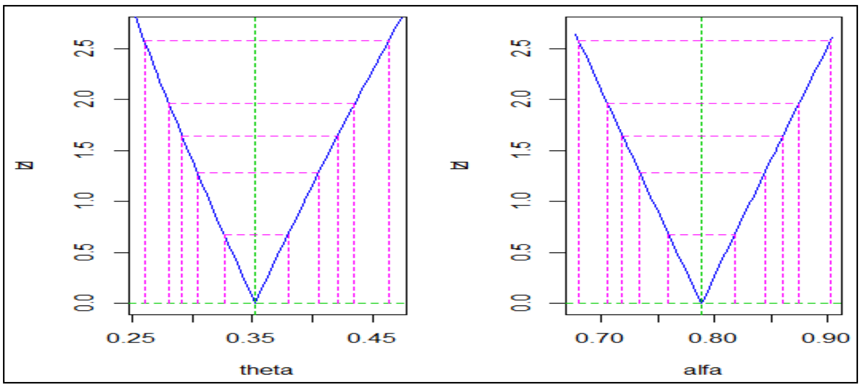

FIGURE 4: Profile of the maximum likelihood estimates of parameters θ and α of PSD.

The fitted plot of the distributions for the considered data set has been shown in the Figure 5.

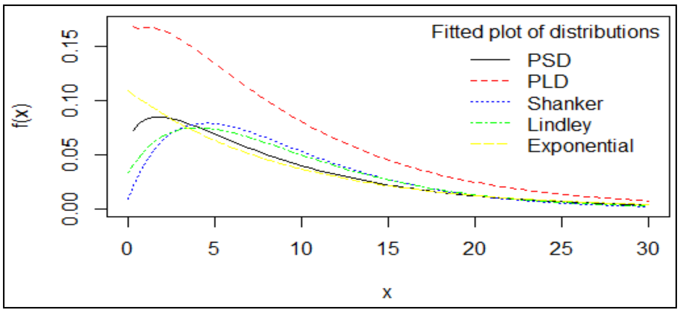

FIGURE 4: Fitted plot of the distributions for the data set.

CONCLUDING REMARKS

In the present paper a two-parameter power Shanker distribution (PSD) which includes Shanker distribution introduced by Shanker (2015) as a particular case has been proposed.[1] Its important statistical properties including shapes of the density, hazard rate function, moments, skewness and kurtosis measures have been discussed. The stochastic ordering of the PSD has been established. Maximum likelihood estimation has been discussed for estimating its parameters. Goodness of fit of PSD has been discussed with a real lifetime data set and the fit shows a quite satisfactory over two-parameter PLD and one parameter Shanker, Lindley and exponential distributions.

Conflict of Interest

Authors declared no conflict of interest or financial support.

Authorship Contributions

Idea/Concept: *Rama Shanker;* ***Design:*** *Rama Shanker;* ***Control/Supervision:*** *Rama Shanker;* ***Data Collection and/or Processing:*** *Rama Shanker;* ***Analysis and/or Interpretation:*** *Kamlesh Kumar Shukla;* ***Literature Review:*** *Kamlesh Kumar Shukla;* ***Writing the Article:*** *Rama Shanker;* ***Critical Review:*** *Rama Shanker;* ***References and Fundings:*** *Rama Shanker;* ***Materials:*** *Kamlesh Kumar Shukla.*

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